Interactive Mathematics Program® (IMP™)

Correlation of *Interactive Mathematics Program (IMP), Years 1–4, Common Core Edition* (2014) to Common Core State Standards (June 2010)

STANDARDS FOR MATHEMATICAL PRACTICE

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up:* adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

Standard	Representative IMP Lessons
1. Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.	Throughout the IMP curriculum, students are assigned Problems of the Week (POWs), which are open-ended problems that cannot be solved easily in a short period of time. POWs help students develop thoughtfulness and perseverance, and force them to focus on their own thinking processes. Students must explain and illustrate their strategies and solutions, and must justify their reasoning in clearly written reports. **Representative IMP Year 1 Lesson:** *Corey Camel* (*The Pit and the Pendulum*) Corey has 3000 bananas to bring to a market that is 1000 miles away. Corey can only carry 1000 bananas per trip and eats one banana for every mile traveled. Out of the 3000 bananas, what is the largest amount of bananas that Corey can bring to market? Initially, students may say "no bananas". But they are asked to make sense of the problem and delve deeper into alternative solutions. To help students make headway to a solution, a mini-Corey Camel problem is presented to them. Students use simulations, pictures, tables of values, and alternative solutions to find an answer. Students work collaboratively, listen to each other's solutions, and prove to each other that their solution is the correct one. **Representative IMP Year 2 Lesson:** *Just Count the Pegs* (*Do Bees Build It Best?*) Students recreate the problem of finding area on a geoboard similar to what confronted Georg Alexander Pick as he formulated his eponymous formula. Students look at different examples of polygons formed on a geoboard, gather data, and construct a formula. Two different approaches are considered and students are asked to support or refute the validity of the two approaches. Students are also encouraged to come up with their own approach

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	as long as they can thoroughly support it. Although an acceptable answer would be Pick's Formula, that is not the point of this POW. Rather, the teacher looks at how students gather data to solve this problem and how they support and defend their own findings while they examine the work of their peers and try to prove them wrong by counterexample. **Representative IMP Year 3 Lesson:** *Let's Make a Deal* (*Pennant Fever*) The classic Monty Hall dilemma from the game show, "Let's Make a Deal" is presented to the students. You can win a great prize or a prize that you could do without. Monty Hall shows you what's behind one of the three curtains and asks you if you would like to change your original curtain or switch. Students are asked which strategy, *switch* or *stay*, is better and why. Students are introduced to the problem through a simulation. Then they are asked to examine the probabilities and mathematics behind the problem. The solution flies in the face of intuition as the probabilities support the "always switch" strategy. Students are asked to explain the better strategy and use mathematics and probability theory to support their choice.
2. Reason abstractly and quantitatively. Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.	**Representative IMP Year 2 Lesson:** *Building the Best Fence* (*Do Bees Build It Best?*) In previous activities, students investigated which dimensions for a rectangular corral yielded the most area. They also investigated whether or not regular polygons had more area than non-regular polygons with the same number of sides. In *Building the Best Fence*, students build upon their prior work to generalize a formula to find the regular polygon that has the most area given the constraint of limited perimeter. **Representative IMP Year 3 Lesson:** *Blue Book* (*Small World, Isn't It?)* High school students often misunderstand depreciation, because they may not have experienced it in their everyday lives. The problem asks students to build a set of data regarding the depreciation of a car over a set number of years. After examining their data, students are asked to generalize a formula to calculate how much a car has depreciated t years after its purchase.

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3. Construct viable arguments and critique the reasoning of others. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.	**Representative IMP Year 3 Lesson:** *Proving Triples* (*Orchard Hideout*) After learning about the Pythagorean Theorem, students find that there are unique sets of numbers that one can find as the measures of the sides of right triangles: Pythagorean triples. Students are asked to examine two sets of measurements and use them to determine if a triangle is a right triangle or not. Students are then asked to write a proof about the multiplication of each member of a Pythagorean triple by a constant and whether or not the results will be the side measurements of a right triangle.

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4. Model with mathematics. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.	**Representative IMP Year 1 Lesson:** *Ox Expressions and Ox Expressions at Home* (*Overland Trail*) Students are given a list of variables and their definition. Students are asked to model real life situations with the variables such as "the amount of water consumed in a day by a family" or "the number of people in a wagon train." The students are also asked to make sense of expressions written with variables. In some cases, the expressions that they are given are nonsense and students must explain why putting two variables together doesn't necessary make for a sensible expression. **Representative IMP Year 3 Lesson:** *Matrices in the Oven* (*Meadows or Malls?*) Given constraints about ingredients used to make various types of cookies, students are asked to model the constraints in the form of matrices. This problem provides a real life context that gives meaning to the rows and columns in a matrix. Students then use their knowledge of matrix operations to find the total amount of each ingredient used. Students must explain how they calculated the numbers they listed in their final matrix.
5. Use appropriate tools strategically. Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.	**Representative IMP Year 1 Lesson:** *Sublett's Cutoff Revisited* (*Overland Trail*) The graphing calculator is used to quickly plot data and allow students to graph various linear functions that will fit the data best. Students have been using paper and pencil methods up to this point and will discover that technology will allow them to explore more efficiently and deeply than their previous experiences. After finding the line of best fit, they use their function to make predictions regarding water consumption over a period of time. **Representative IMP Year 2 Lesson:** *Parabolas and Equations I and III* (*Fireworks*) Students use the graphing calculator to explore families of functions. They begin with the simple function for a parabola and then investigate what parts of the function make the graph narrower, wider, inverted, and translated in any direction. Using the investigative approach, students soon discover what each parameter does in $y = a(x)^2 + k$

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6. Attend to precision. Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.	**Representative IMP Year 1 Lesson:** *You're the Storyteller: From Rules to Situations* (*Overland Trail*) Students are given a set of equations and must create a context that the equation could represent. Precision is important in this activity, as students must create a situation, clearly state what the variable represents (including units), and then find the number that will make the given equation true. Clear communication of their variables and their meaning is vital to this activity. **Representative IMP Year 2 Lesson:** *How Many More People?* (*Small World, Isn't it?)* In order to explore population growth, students must graphically represent population data over time. Students are asked to graph this data on an appropriate scale of axes. Using their graph or using algebra, they then find the average increases over different periods of time. They are then asked to look at different intervals of time in order to compare growth rates. Precision in graphing and calculating the average increase is used throughout the activity.
7. Look for and make use of structure. Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.	**Representative IMP Year 1 Lesson:** *Continuing the Pattern* (*All About Alice*) Students explore the various powers of two in this activity. Throughout this unit, they have worked with positive exponents and see the growth with a base of two. They now explore negative exponents and examine the various patterns that emerge with negative exponents. Students explain how to find the result of a negative exponent using fractions as their results. **Representative IMP Year 3 Lesson:** *Which is Which?* and *Formulas for* ${}_nP_r$ *and* ${}_nC_r$ (*Pennant Fever*) Combinations and permutations are explored in these activities. Students examine their previous work with combinations and permutations and are asked to explain the difference between the two. They must use the proper notation for both. After looking at the patterns in their work, students find a general formula for permutations in terms of n and r. They are also asked to find a general equation expressing the relationship between permutations and combinations.

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8. Look for and express regularity in repeated reasoning. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.	**Representative IMP Year 2 Lesson:** *Don't Fence Me In; More Fencing, Bigger Corrals;* and *Building the Best Fence* (*Do Bees Build It Best?*) Students build to the general formula to find the area of any regular polygon by investigating these three activities. Much like their experience in Year 1, they begin with a simple polygon (a quadrilateral) and work their way through polygons of increasing number of sides. By examining their work with these various polygons, students derive the area formula for a regular polygon. **Representative IMP Year 3 Lesson:** *Squaring the Circle; Using the Squared Circle; Hexagoning the Circle; Octagoning the Circle;* and *Polygoning the Circle* (*Orchard Hideout*) Students explore the case of the circumscribed polygon about a circle. Beginning with the square and working up to a polygon with *n* sides, students calculate the perimeter and area of the circumscribed polygon in terms of the radius of the circle. After examining their work, students will generalize a formula for the perimeter and area of any sided polygon.

Number and Quantity

Standard	*IMP* Lessons
The Real Number System	
Extend the properties of exponents to rational exponents.	
1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*	***IMP Year 1, All About Alice:*** A Half Ounce of Cake, 442 It's in the Graph, 443 Stranger Pieces of Cake, 446 All Roads Lead to Rome, 449
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.	***IMP Year 1, All About Alice:*** Stranger Pieces of Cake, 446 All Roads Lead to Rome, 449
Use properties of rational and irrational numbers.	
3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	***IMP Year 1, All About Alice:*** Rational versus Irrational, 476
Quantities*	
Reason quantitatively and use units to solve problems.	
1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	***IMP Year 1, The Overland Trail:*** In Need of Numbers, 46 The Search for Dry Trails, 28 Previous Travelers, 55 Who Will Make It?, 59 This standard is addressed throughout IMP Years 1–4.
2. Define appropriate quantities for the purpose of descriptive modeling.	***IMP Year 1, The Overland Trail:*** Ox Expressions, 36 Ox Expressions at Home, 38 Travel on the Trail, 72 Moving Along, 74 This standard is addressed throughout IMP Years 1–4.

NUMBER AND QUANTITY

Standard	*IMP* Lessons
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	***IMP Year 1, The Pit and the Pendulum:*** Building a Pendulum, 142 Time is Relative, 146 What's Your Stride, 147 *Penny Weight* Revisited, 176 An Important Function, 201 ***IMP Year 2, Do Bees Build it Best?:*** Falling Bridges, 329
The Complex Number System	
Perform arithmetic operations with complex numbers.	
1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.	***IMP Year 3, High Dive:*** Imagine a Solution, 268 Complex Numbers and Quadratic Equations, 270
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	***IMP Year 3, High Dive:*** Complex Components, 271
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	***IMP Year 3, High Dive:*** Complex Conjugation, 308
Represent complex numbers and their operations on the complex plane.	
4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	***IMP Year 3, High Dive:*** Complex Components, 271
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(1 - \sqrt{3}i)^3 = 8$ *because* $(1 - \sqrt{3}i)$ *has modulus 2 and argument 120°.*	***IMP Year 3, High Dive:*** Complex Conjugation, 308

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Standard	*IMP* Lessons
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	***IMP Year 3, High Dive:*** Absolutely Complex, 309
Use complex numbers in polynomial identities and equations.	
7. Solve quadratic equations with real coefficients that have complex solutions.	***IMP Year 3, High Dive:*** Complex Numbers and Quadratic Equations, 270
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$.	***IMP Year 3, High Dive:*** Finding with the Formula, 266 Using Your ABCs, 267 Imagine a Solution, 268 Complex Numbers and Quadratic Equations, 270
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	***IMP Year 3, High Dive:*** The Fundamental Theorem of Algebra, 314
Vector and Matrix Quantities	
Represent and model with vector quantities.	
1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., ***v***, |***v***|, ||***v***||, ***v***).	***IMP Year 3, High Dive:*** Absolutely Complex, 309
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	***IMP Year 3, High Dive:*** Swimming Pointers, 282
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.	***IMP Year 3, High Dive:*** Vector Velocities, 283 Swimming Pointers, 282
Perform operations on vectors.	
4. (+) Add and subtract vectors.	***IMP Year 3, High Dive:*** Complex Components, 271

Number and Quantity	
Standard	***IMP* Lessons**
4a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.	***IMP Year 3, High Dive:*** Complex Components, 271 Vector Island, 320
4b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.	***IMP Year 3, High Dive:*** Vector Velocities, 283
4c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	***IMP Year 3, High Dive:*** Vector Velocities, 283
5. (+) Multiply a vector by a scalar.	***IMP Year 3, High Dive:*** Scalars and Magnitude, 322
5a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$.	***IMP Year 3, High Dive:*** Scalars and Magnitude, 322
5b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $\|\|c\boldsymbol{v}\|\| = \|c\|\boldsymbol{v}$. Compute the direction of $c\boldsymbol{v}$ knowing that when $\|c\|\boldsymbol{v} \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).	***IMP Year 3, High Dive:*** Vector Island, 320 Scalars and Magnitude, 322
Perform operations on matrices and use matrices in applications.	
6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	***IMP Year 4, Meadows or Malls?:*** Inventing an Algebra, TBD Flying Matrices, TBD
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	***IMP Year 4, Meadows or Malls?:*** Fresh Ingredients, TBD
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.	***IMP Year 4, Meadows or Malls?:*** Inventing an Algebra, TBD Back and Forth, TBD

Number and Quantity

Standard	*IMP* Lessons
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	***IMP Year 4, Meadows or Malls?:*** Things We Take for Granted, TBD
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.	***IMP Year 4, Meadows or Malls?:*** Solving the Simplest, TBD
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	***IMP Year 4, As the Cube Turns:*** Translation in Two Dimensions, TBD Rotation in Two Dimensions, TBD
12. (+) Work with 2 × 2 matrices as a transformations of the plane, and interpret the absolute value of the determinant in terms of area.	***IMP Year 4, As the Cube Turns:*** Translation in Two Dimensions, TBD Rotation in Two Dimensions, TBD

Algebra

Standard	*IMP* Lessons
Seeing Structure in Expressions	
Interpret the structure of expressions.	
1. Interpret expressions that represent a quantity in terms of its context.★	***IMP Year 1, The Overland Trail:*** Ox Expressions, 36 Ox Expressions at Home, 38 This standard is addressed throughout IMP Years 1–4.
1a. Interpret parts of an expression, such as terms, factors, and coefficients.	***IMP Year 1, The Overland Trail:*** Moving Along, 74 Fair Share on Chores, 79 This standard is addressed throughout IMP Years 1–4.

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1b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.*	***IMP Year 1, The Pit and the Pendulum:*** *Penny Weight* Revisited, 176 The Best Spread, 170 Making Friends with Standard Deviation, 171 Deviations, 173 ***IMP Year 3, High Dive:*** Planning for Formulas, 237
2. Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.*	***IMP Year 1, The Overland Trail:*** More Scrambled Equations and Mystery Bags, 95 ***IMP Year 2, Fireworks:*** Square It!, 26 Vertex Form Challenge, 72 Factors of Research, 74
Write expressions in equivalent forms to solve problems.	
3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.	***IMP Year 1, The Overland Trail:*** More Fair Share on Chores, 82 More Scrambled Equations and Mystery Bags, 95
3a. Factor a quadratic expression to reveal the zeros of the function it defines.	***IMP Year 2, Fireworks:*** Factoring, 47 Let's Factor!, 48 Solve That Quadratic!, 49 Quadratic Challenge, 76
3b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.	***IMP Year 2, Fireworks:*** Squares and Expansions, 27 Here Comes Vertex Form, 36 Another Rocket, 38
3c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*	***IMP Year 1, All About Alice:*** A Wonderland Lost, 427 Inflation, Depreciation, and Alice, 463 ***IMP Year 2, Small World, Isn't It?:*** The Generous Banker, 449 The Limit of Their Generosity, 452

Algebra	
Standard	***IMP* Lessons**
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.*	***IMP Year 1, All About Alice:*** More About Rallods, 466 ***IMP Year 2, Small World, Isn't It?:*** Summing the Sequences—Part II, 474
Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	
1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	***IMP Year 2, Fireworks:*** A Summary of Quadratics and Other Polynomials, 54
Understand the relationship between zeros and factors of polynomials.	
2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.	***IMP Year 3, The World of Functions:*** The Arithmetic of Polynomials II—Factors, 419
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	***IMP Year 2, Fireworks:*** Make Your Own Intercepts, 75
Use polynomial identities to solve problems.	
4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*	***IMP Year 3, The World of Functions:*** The Arithmetic of Polynomials I—Operations, 417 The Arithmetic of Polynomials III—Powers, 421
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.	***IMP Year 3, Pennant Fever:*** The Binomial Theorem and Row Sums, 92 The Whys of Binomial Expansion, 91

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Standard	***IMP* Lessons**
Rewrite rational expressions.	
6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.	***IMP Year 3, The World of Functions:*** The Arithmetic of Polynomials I—Operations, 417
7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	***IMP Year 3, The World of Functions:*** The Arithmetic of Polynomials I—Operations, 417
Creating Equations*	
Create equations that describe numbers or relationships.	
1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*	***IMP Year 1, The Overland Trail:*** The Mystery Bags Game, 90 More Mystery Bags, 92 ***IMP Year 1, All About Alice:*** Many Meals for Alice, 433 ***IMP Year 1, Cookies:*** How Many of Each Kind?, 344 This standard is addressed throughout IMP Years 1–4.

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2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	***IMP Year 1, The Overland Trail:*** More Fair Share for Hired Hands, 83 The Big Buy, 85 Getting the Gold, 88 Keeping Track, 125 Westville Formulas, 129 ***IMP Year 1, The Pit and The Pendulum:*** The Thirty-Foot Prediction, 203 ***IMP Year 1, Cookies:*** A Charity Rock, 391 Big State U, 393 ***IMP Year 2, Fireworks:*** Pens and Corrals in Vertex Form, 40
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*	***IMP Year 1, Cookies:*** Profitable Pictures, 367 You Are What You Eat, 374 ***IMP Year 4, Meadows or Malls?:*** Eastside Westside Story, TBD
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*	***IMP Year 1, The Overland Trail:*** Fair Share on Chores (#4), 79 Fair Share for Hired Hands (#5), 81 ***IMP Year 1, Shadows:*** More Triangles for Shadows, 302 ***IMP Year 1, Cookies:*** Getting on Good Terms, 378 ***IMP Year 2, Fireworks:*** Crossing the Axis, 17

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Standard	***IMP* Lessons**
Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	
1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	***IMP Year 1, The Overland Trail:*** The Mystery Bags Game, 90 More Mystery Bags, 92 Scrambling Equations, 93 More Scrambled Equations and Mystery Bags, 95 ***IMP Year 2, Fireworks:*** Quadratic Choices, 50
2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	***IMP Year 2, Do Bees Build it Best:*** Simply Square Roots, 327
Solve equations and inequalities in one variable.	
3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	***IMP Year 1, The Overland Trail:*** More Fair Share for Hired Hands, 83 More Mystery Bags, 92 More Scrambled Equations and Mystery Bags, 95 ***IMP Year 1, Cookies:*** Only One Variable, 382 Investigating Inequalities, 347 My Simplest Inequality, 349
4. Solve quadratic equations in one variable.	***IMP Year 2, Fireworks:*** Another Rocket, 38 Profiting from Widgets, 39 Solve That Quadratic!, 49 Quadratic Challenge, 76
4a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.	***IMP Year 2, Fireworks:*** Squares and Expansions, 27 The Quadratic Formula, 70

ALGEBRA	
Standard	***IMP* Lessons**
4b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.	***IMP Year 2, Fireworks:*** Square It!, 26 Squares and Expansions, 27 Let's Factor!, 48 Solve That Quadratic!, 49 The Quadratic Formula, 70 ***IMP Year 2, Do Bees Build It Best?:*** Impossible Rugs, 312
Solve systems of equations.	
5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	***IMP Year 1, Cookies:*** The Classic Way to Get the Point, 417
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	***IMP Year 1, The Overland Trail:*** Family Comparisons by Algebra, 97 ***IMP Year 1, Cookies:*** Going Out For Lunch, 379 Get the Point, 381 Set It Up, 383 A Reflection on Money, 384
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.	***IMP Year 1, Cookies:*** Algebra Pictures, 407 ***IMP Year 2, Fireworks:*** Another Rocket, 38
8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.	***IMP Year 4, Meadows or Malls?:*** Inventing an Algebra, TBD
9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).	***IMP Year 4, Meadows or Malls?:*** Finding an Inverse, TBD Inverses and Equations, TBD

ALGEBRA	
Standard	***IMP* Lessons**
Represent and solve equations and inequalities graphically.	
10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	***IMP Year 1, The Overland Trail:*** From Rules to Graphs, 51 Graphing Calculator In-Outs, 64 ***IMP Year 1, The Pit and The Pendulum:*** Graphs in Search of Equations I, 197 Graphs in Search of Equations II, 198 Graphs in Search of Equations III, 202 ***IMP Year 1, Cookies:*** Picturing Cookies—Part II, 360 Picturing Pictures, 365
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★	***IMP Year 1, The Overland Trail:*** Sublette's Cutoff, 58 Fort Hall Businesses, 66 ***IMP Year 1, The Pit and the Pendulum:*** Graphing Free-for-All, 196 Graphs in Search of Equations I, 197 Graphs in Search of Equations II, 198 Graphing Summary, 199 ***IMP Year 1, Cookies:*** Get The Point, 381 Set It Up, 383 ***IMP Year 2, Fireworks:*** Pens and Corrals in Vertex Form, 40 Coming Down, 44 This standard is addressed throughout IMP Years 1–4.
12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	***IMP Year 1, Cookies:*** Picturing Cookies—Part I, 354 Picturing Cookies—Part II, 360 Picturing Pictures, 365

FUNCTIONS	
Standard	***IMP* Lessons**
Interpreting Functions	
Understand the concept of a function and use function notation.	
1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	***IMP Year 1, The Overland Trail:*** Inside Out, 10 Pulling Out Rules, 12 To Kearny by Equation, 33 From Rules to Graphs, 51
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	***IMP Year 1, Overland Trail:*** All Four, One—Linear Functions, 76 Moving Along, 74 Water Conservation, 84 The Big Buy, 85 Getting the Gold, 88 ***IMP Year 2, Fireworks:*** Victory Celebration, 4 Another Rocket, 38 Profiting from Widgets, 39 Fireworks in the Sky, 43
3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.	***IMP Year 1, The Overland Trail:*** Keep It Going, 104 ***IMP Year 1, All About Alice:*** Rallods in Rednow Land, 437 More About Rallods, 466

FUNCTIONS	
Standard	***IMP* Lessons**
Interpret functions that arise in applications in terms of the context.	
4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★	***IMP Year 1, The Overland Trail:*** Wagon Train Sketches and Situations, 43 Graph Sketches, 45 Family Comparisons by Algebra, 97 ***IMP Year 1, All About Alice:*** It's in the Graph, 443 ***IMP Year 1, Cookies:*** Big State U, 393 ***IMP Year 2, Fireworks:*** Victory Celebration, 4 This standard is addressed throughout IMP Years 1–4.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★	***IMP Year 1, The Overland Trail:*** Travel on the Trail, 72 Fair Share on Chores, 79 *The Basic Student Budget* Revisited, 68 ***IMP Year 1, All About Alice:*** It's in the Graph, 443 ***IMP Year 2, Fireworks:*** A Corral Variation, 10
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★	***IMP Year 1, The Overland Trail:*** Moving Along, 74 Travel on the Trail, 72 Following Families on the Trail, 62

FUNCTIONS	
Standard	***IMP* Lessons**
Analyze functions using different representations.	
7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★	***IMP Year 1, The Overland Trail:*** Straight-Line Reflections, 77 ***IMP Year 1, The Pit and the Pendulum:*** Graphing Free-for-All, 196 ***IMP Year 1, All About Alice:*** It's in the Graph, 443 Exponential Graphing, 469 ***IMP Year 2, Fireworks:*** Victory Celebration, 4
7a. Graph linear and quadratic functions and show intercepts, maxima, and minima.	***IMP Year 1, The Overland Trail:*** The Growth of Westville, 128 ***IMP Year 1, Cookies:*** Finding Linear Graphs, 370 ***IMP Year 2, Fireworks:*** Victory Celebration, 4 Crossing the Axis, 17 Here Comes Vertex Form, 36 Finding Vertices and Intercepts, 37 This standard is addressed throughout IMP Years 1–4.
7b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.	***IMP Year 1, The Overland Trail:*** Wagon Train Sketches and Situations (piecewise-defined), 43 Integers Only (step function), 113 ***IMP Year 1, The Pit and the Pendulum:*** Graphing Summary, 199 The Thirty-Foot Prediction, 203 Maybe It's a Cube Root!, 215 Another Vertex? Absolutely!, 227 Piecewise Functions, 234
7c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.	***IMP Year 2, Fireworks:*** Another Rocket, 38 ***IMP Year 3, The World of Functions:*** The End of the Function, 348

Functions	
Standard	***IMP* Lessons**
7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	***IMP Year 3, The World of Functions:*** Approaching Infinity, 347
7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	***IMP Year 1, All About Alice:*** Graphing Alice, 426 A Wonderland Lost, 427 It's in the Graph, 443 Taking Logs to the Axes, 455 Exponential Graphing, 469 ***IMP Year 3, High Dive:*** Sand Castles, 208
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.	***IMP Year 2, Fireworks:*** Parabolas and Equations I, 11 Parabolas and Equations II, 12 Parabolas and Equations III, 14 Here Comes Vertex Form, 36 Vertex Form for Parabolas, 15
8a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.	***IMP Year 2, Fireworks:*** Finding Vertices and Intercepts, 37 Pens and Corrals in Vertex Form, 40
8b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*	***IMP Year 1, All About Alice:*** A Wonderland Lost, 427 Inflation, Depreciation, and Alice, 463 ***IMP Year 2, Small World, Isn't It?:*** Return to *A Crowded Place*, 457
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*	***IMP Year 2, Fireworks:*** Quadratics Choices, 50 A Quadratic Summary, 51

FUNCTIONS	
Standard	***IMP* Lessons**
Building Functions	
Build a function that models a relationship between two quantities.	
1. Write a function that describes a relationship between two quantities.★	This standard is addressed throughout IMP Years 1–4.
1a. Determine an explicit expression, a recursive process, or steps for calculation from a context.	This standard is addressed throughout IMP Years 1–4.
1b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*	***IMP Year 3, The World of Functions:*** The Cost of Pollution, 375 Slide That Function, 390
1c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*	***IMP Year 3, The World of Functions:*** Order Among the Functions, 377 Cozying Up to Composition, 378
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★	***IMP Year 1, The Overland Trail:*** What's Next?, 4 Diagonals Illuminated, 105 ***IMP Year 2, Small World, Isn't It?:*** POW 12: Planning the Platforms, 408

FUNCTIONS

Standard	*IMP* Lessons
Build new functions from existing functions.	
3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	***IMP Year 1, The Pit and the Pendulum:*** Graphing Free-for-All, 196 ***IMP Year 2, Fireworks:*** Parabolas and Equations I, 11 Parabolas and Equations II, 12 Parabolas and Equations III, 14
4. Find inverse functions.	***IMP Year 1, All About Alice:*** Alice on a Log, 454 Taking Logs to the Axes, 455 ***IMP Year 3, The World of Functions:*** An Inventory of Inverses, 385
4a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *for* $x > 0$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.	***IMP Year 3, The World of Functions:*** Linear Functions in Verse, 384
4b. (+) Verify by composition that one function is the inverse of another.	***IMP Year 3, The World of Functions:*** An Inventory of Inverses, 385
4c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.	***IMP Year 3, The World of Functions:*** An Inventory of Inverses, 385
4d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	***IMP Year 3, The World of Functions:*** No Inverse? Restrict the Outputs!, 416
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.	***IMP Year 1, All About Alice:*** Alice on a Log, 454 Taking Logs to the Axes, 455

FUNCTIONS	
Standard	***IMP*** **Lessons**
Linear and Exponential Models	
Construct and compare linear and exponential models and solve problems.	
1. Distinguish between situations that can be modeled with linear functions and with exponential functions.	***IMP Year 1, All About Alice:*** Alice in Wonderland, 422 Graphing Alice, 426
1a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.	***IMP Year 1, The Overland Trail:*** Following Families on the Trail, 62 ***IMP Year 1, All About Alice:*** A Wonderland Lost, 427 A New Kind of Cake, 430
1b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.	***IMP Year 1, The Overland Trail:*** Fort Hall Businesses, 66
1c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	***IMP Year 1, All About Alice:*** A Wonderland Lost, 427 ***IMP Year 2, Small World, Isn't It?*** Comparative Growth, 397
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	***IMP Year 1, The Pit and the Pendulum:*** So Little Data, So Many Rules, 195 ***IMP Year 2, Small World, Isn't It?:*** POW 12: Planning the Platforms, 408
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	***IMP Year 1, All About Alice:*** Rallods in Rednow Land, 437
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	***IMP Year 2, Small World, Isn't It?:*** Return to *A Crowded Place*, 457

FUNCTIONS	
Standard	***IMP* Lessons**
Interpret expressions for functions in terms of the situation they model.	
5. Interpret the parameters in a linear or exponential function in terms of a context.	***IMP Year 1, The Overland Trail:*** Water Conservation, 84 ***IMP Year 1, All About Alice:*** Measuring Meals for Alice, 451
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	
1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	***IMP Year 4, How Much? How Fast?*** Trying a New Angle, TBD Different Angles, TBD
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	***IMP Year 3, High Dive:*** Extending the Sine, 201 What's Your Cosine?, 233
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.	***IMP Year 4, How Much? How Fast?:*** Different Angles, TBD
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	***IMP Year 4, How Much? How Fast?:*** Different Angles, TBD
Model periodic phenomena with trigonometric functions.	
5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★	***IMP Year 3, High Dive:*** Graphing the Ferris Wheel, 204 Ferris Wheel Graph Variations, 205
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	***IMP Year 3, The World of Functions:*** No Inverse? Restrict the Outputs!, 416

FUNCTIONS

Standard	*IMP* Lessons
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★	***IMP Year 3, High Dive:*** Not So Spectacular, 219 A Practice Jump, 220
Prove and apply trigonometric identities.	
8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to calculate trigonometric ratios.	***IMP Year 3, High Dive:*** Pythagorean Trigonometry, 242 More Pythagorean Trigonometry, 299
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.	***IMP Year 4, As the Cube Turns:*** The Sine of a Sum, TBD Sum Tangents, TBD

GEOMETRY

Standard	*IMP* Lessons
Congruence	
Experiment with transformations in the plane	
1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	***IMP Year 1, Shadows:*** More About Angles, 286 ***IMP Year 3, Orchard Hideout:*** The Distance Formula, 109 Defining Circles, 136 ***IMP Year 2, Geometry by Design:*** What Do We Already Know?, 86 Circle and Arc Designs—Introducing the Compass, 87

GEOMETRY

Standard	*IMP* Lessons
2. Model transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus stretch in a specific direction).	***IMP Year 2, Geometry by Design:*** Reflection Challenges, 138 Reflecting Lines, 140 Rotation with Coordinates, 147 Perpendicular Rotations, 149 Translation Investigations, 153 ***IMP Year 4, As the Cube Turns:*** Flipping Points, TBD
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	***IMP Year 2, Geometry by Design:*** Isometric Transformation 2: Rotation, 142 ***IMP Year 4, As the Cube Turns:*** Further Flips, TBD
4. Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments.	***IMP Year 2, Geometry by Design:*** Isometric Transformation 1: Reflection, 136 Isometric Transformation 2: Rotation, 142 Isometric Transformation 3: Translation, 150 ***IMP Year 4, As the Cube Turns:*** An Animated POW Write-up, TBD
5. Given a specified rotation, reflection or translation and a geometric figure, construct the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Construct a sequence of transformations that will carry a given figure onto another.	***IMP Year 2, Geometry by Design:*** Reflection Challenges, 138 Isometric Transformation 2: Rotation, 142 How Many Ways from *A* to *B*?, 145 Rotation with Coordinates, 147 Frieze Frame, 151 Transforming One Shape Into Another, 155 Drawing Conclusions, 168 Combinations of Transformations, 171 ***IMP Year 4, As the Cube Turns:*** Further Flips, TBD
Understand congruence in terms of rigid motions	
6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a rigid motion on a figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	***IMP Year 1, Shadows:*** Are Angles Enough?, 279 ***IMP Year 2, Geometry by Design:*** What Makes a Triangle?, 104

GEOMETRY	
Standard	***IMP*** **Lessons**
7. Explain using rigid motions the meaning of congruence for triangles as the equality of all corresponding pairs of sides and all corresponding pairs of angles.	***IMP Year 1, Shadows:*** Why Are Triangles Special?, 277 Are Angles Enough?, 279
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence.	***IMP Year 2, Geometry by Design:*** Drawing Triangles with Ruler and Protractor—SAS and ASA, 95 Drawing Specific Triangles—SSS, 100 What Makes a Triangle?, 104
Prove geometric theorems	
9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*	***IMP Year 1, Shadows:*** More About Angles, 286 Inside Similarity, 289 A Parallel Proof, 290 ***IMP Year 2, Geometry by Design:*** Postulates, Axioms, and Our First Theorem, 88 Why It Works: The Perpendicular Bisector Construction, 121 Angle Bisector Explorations, 127 Parallel Lines and Transversals, 130 ***IMP Year 3, Orchard Hideout:*** POW 5: Equally Wet, 99
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*	***IMP Year 1, Shadows:*** Triangles Versus Other Polygons, 274 Very Special Triangles, 283 ***IMP Year 2, Geometry by Design:*** Proving the Isosceles Triangle Theorem, 111
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangle are parallelograms with congruent diagonals.*	***IMP Year 1, Shadows:*** Angles, Angles, Angles, 291 Fit Them Together, 334 ***IMP Year 2, Geometry by Design:*** Proofs for You!, 116 Properties of Special Quadrilaterals, 129 Parallelogram and Trapezoid Proofs, 133 ***IMP Year 3, Orchard Hideout:*** Midpoint Quadrilaterals, 161

GEOMETRY	
Standard	***IMP* Lessons**
Make geometric constructions	
12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*	***IMP Year 2, Geometry by Design:*** Construct an Equilateral Triangle, 91 Circle Designs, 92 Drawing Specific Triangles—SSS, 100 Perpendicular Bisector Explorations, 119 Angle Bisector Explorations, 127 Construction Challenges, 123 ***IMP Year 3, Orchard Hideout:*** POW 5: Equally Wet, 99 Only Two Flowers, 101 A Perpendicularity Proof, 102 POW 6: On Patrol, 116
13. Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle.	***IMP Year 2, Geometry by Design:*** Construct an Equilateral Triangle, 91 Hexagon Designs, 178 Construction Challenges, 123
Similarity, Right Triangles, and Trigonometry	
Understand similarity in terms of similarity transformations	
1. Verify experimentally the properties of dilations:	***IMP Year 1, Shadows:*** Draw the Same Shape, 258 How to Shrink It?, 260 Make It Similar, 262 ***IMP Year 2, Geometry by Design:*** A Non-Isometric Transformation: Dilation, 158
1a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.	***IMP Year 1, Shadows:*** Draw the Same Shape, 258 How to Shrink It?, 260 ***IMP Year 2, Geometry by Design:*** Think About It, 166

Geometry	
Standard	***IMP* Lessons**
1b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	***IMP Year 1, Shadows:*** Draw the Same Shape, 258 How to Shrink It?, 260 The Statue of Liberty's Nose, 261 ***IMP Year 2, Geometry by Design:*** Dilating a Right Triangle, 160
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all pairs of angles and the proportionality of all pairs of sides.	***IMP Year 1, Shadows:*** Ins and Outs of Proportion, 267 Similar Problems, 269
3. Use the properties of similarity transformations to establish the AA criterion for similarity of triangles.	***IMP Year 1, Shadows:*** Triangles Versus Other Polygons, 274
Prove theorems involving similarity	
4. Prove theorems about triangles using similarity transformations. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean theorem proved using triangle similarity.*	***IMP Year 1, Shadows:*** Inside Similarity, 289 ***IMP Year 2, Do Bees Build It Best?;*** Pythagorean Proof, 366 Pythagoras by Proportion, 368
5. Use triangle congruence and similarity criteria to solve problems and to prove relationships in geometric figures.	***IMP Year 1, Shadows:*** Mirror Madness, 296 A Shadow of a Doubt, 297 ***IMP Year 2, Geometry by Design:*** Dilation Investigations, 163 Congruent Triangles Proofs, 182 Parallel Lines and Transversals, 130
Define trigonometric ratios and solve problems involving right triangles	
6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	***IMP Year 1, Shadows:*** Right Triangle Ratios, 305 Homemade Trig Tables, 309

GEOMETRY

Standard	*IMP* Lessons
7. Explain and use the relationship between the sine and cosine of complementary angles.	***IMP Year 1, Shadows:*** Your Opposite Is My Adjacent, 310
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	***IMP Year 1, Shadows:*** The Tree and the Pendulum, 311 Sparky and the Dude, 312 ***IMP Year 2, Bees:*** Leslie's Floral Angles, 330
(+) Apply trigonometry to general triangles	
9. Derive the formula $A = ½ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	***IMP Year 2, Do Bees Build It Best?;*** Another Area Formula, 374
10. Prove the Laws of Sines and Cosines and use them to solve problems.	***IMP Year 2, Do Bees Build It Best?;*** Beyond Pythagoras, 370 Comparing Sines, 371 ***IMP Year 2, Geometry by Design:*** The Law Is On Your Side, 186
11. Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	***IMP Year 2, Do Bees Build It Best?;*** Beyond Pythagoras, 370 Comparing Sines, 371 ***IMP Year 2, Geometry by Design:*** The Law Is On Your Side, 186
Circles	
Understand and apply theorems about circles	
1. Prove that all circles are similar.	***IMP Year 2, Geometry by Design:*** Think About It, 166
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*	***IMP Year 3, Orchard Hideout:*** Inscribed Angles, 157 More Inscribed Angles, 158 Angles In and Out, 159

GEOMETRY	
Standard	***IMP* Lessons**
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	***IMP Year 3, Orchard Hideout*** The Inscribed Circle, 167 Medians and Altitudes, 168
4. (+) Construct a tangent line from a point outside a given circle to the circle.	***IMP Year 3, Orchard Hideout:*** Constructing Tangents
Find arc lengths and areas of sectors of circles	
5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	***IMP Year 3, Orchard Hideout:*** Goat on a Rope, 160 ***IMP Year 3, High Dive:*** Trying a New Angle, 308 Different Angles, 310
Expressing Geometric Properties with Equations	
Translate between the geometric description and the equation for a conic section	
1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	***IMP Year 3, Orchard Hideout:*** Completing the Square and Getting a Circle, 139
2. Derive the equation of a parabola given a focus and directrix.	***IMP Year 3, Orchard Hideout:*** What's A Parabola?, 175
3. (+) Derive the equations of ellipses and hyperbolas given two foci for the ellipse, and two directrices of a hyperbola.	***IMP Year 3, Orchard Hideout:*** Ellipses and Hyperbola by Points and Algebra, 183
Use coordinates to prove simple geometric theorems algebraically	
4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).*	***IMP Year 3, Orchard Hideout:*** Proving with Distance—Part I, 113 Proving with Distance—Part II, 118

GEOMETRY

Standard	*IMP* Lessons
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	***IMP Year 2, Geometry by Design:*** Sloping Slides, 148 Perpendicular Rotations, 149
6. Find the point on a directed line segment between two given points that divide the segment in a given ratio.	***IMP Year 1, Shadows:*** POW 11: Trying Triangles, 287
7. Use coordinates to compute perimeters of polygons and areas for triangles and rectangles, e.g. using the distance formula.★	***IMP Year 3, Orchard Hideout:*** Sprinkler in the Orchard, 108 Daphne's Dance Floor, 134
Geometric Measurement and Dimension	
Explain volume formulas and use them to solve problems	
1. Give an informal argument for the formulas for the volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*	***IMP Year 2, Do Bees Build It Best?:*** Which Holds More?, 348 Back on the Farm, 347 Shedding Light on Prisms, 344 ***IMP Year 3, Orchard Hideout:*** More Volume Formulas, 172
2. (+) Given an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	***IMP Year 3, Orchard Hideout:*** More Volume Formulas, 172
3. Use volume formulas for cylinders, pyramids, cones and spheres to solve problems.★	***IMP Year 2, Do Bees Build It Best?:*** Back on the Farm, 347 ***IMP Year 3, Orchard Hideout:*** Cylindrical Soda, 143 More Volume Formulas, 172 Knitting, 174
Visualize relationships between two-dimensional and three-dimensional objects	
4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	***IMP Year 2, Do Bees Build It Best?:*** Flat Cubes, 334 A Voluminous Task, 337 ***IMP Year 4, As the Cube Turns:*** An Animated POW, TBD

Geometry

Standard	*IMP* Lessons
Modeling with Geometry	
Apply geometric concepts in modeling situations	
1. Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★	***IMP Year 3, Orchard Hideout:*** Orchard Growth Revisited, 130
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★	***IMP Year 2, Small World, Isn't It?:*** What a Mess!, 394 The Growth of the Oil Slick, 420
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy constraints or minimize cost; working with typographic grid systems based on ratios).★	***IMP Year 2, Do Bees Build it Best?:*** Not a Sound, 336 POW 10: Possible Patches, 332

Statistics and Probability

Standard	*IMP* Lessons
Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on a single count or measurement variable	
1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	***IMP Year 2, The Game of Pig:*** What Are the Chances?, 206 Rollin', Rollin', Rollin', 217 Waiting for a Double, 201
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	***IMP Year 1, The Pit and the Pendulum:*** Data Spread, 163 The Best Spread, 170
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	***IMP Year 1, The Pit and the Pendulum:*** Making Friends with Standard Deviation, 171 Deviations, 173

STATISTICS AND PROBABILITY	
Standard	***IMP* Lessons**
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve.	***IMP Year 1, The Pit and the Pendulum:*** *Penny Weight* Revisited, 176 Can Your Calculator Pass This Soft Drink Test?, 177 Standard Deviation Basics, 166
Summarize, represent, and interpret data on two categorical and quantitative variables	
5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.	***IMP Year 1, The Overland Trail:*** Categorical Data on the Wagon Train, 132 ***IMP Year 3, Is There Really a Difference?:*** What Would You Expect?, 477 Who's Absent?, 479 Big and Strong, 481
6. Represent data on two quantitative variables on a scatter plot and describe how the variables are related.	***IMP Year 1, The Overland Trail:*** Previous Travelers, 55 Sublette's Cutoff, 58
6a. Use a model function fitted to the data to solve problems in the context of the data. *Use given model functions or choose a function suggested by the context. Emphasize linear and exponential models.*	***IMP Year 1, The Overland Trail:*** Who Will Make It?, 59 The Basic Student Budget, 60 ***IMP Year 2, Small World, Isn't It?:*** California and Exponents, 446
6b. Informally assess the fit of a model function by plotting and analyzing residuals.	***IMP Year 1, The Pit and the Pendulum:*** Could It Be Linear?, 213
6c. Fit a linear function for scatter plots that suggest a linear association.	***IMP Year 1, The Overland Trail:*** Previous Travelers, 55 *Sublette's Cutoff* Revisited, 67
Interpret linear models	
7. Interpret the slope (rate of change) and the intercept (constant term) of a linear fit in the context of the data.	***IMP Year 1, The Overland Trail:*** Fort Hall Businesses, 66 Moving Along, 74
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.	***IMP Year 1, The Pit and the Pendulum:*** Could It Be Linear?, 213

STATISTICS AND PROBABILITY	
Standard	***IMP* Lessons**
9. Distinguish between correlation and causation.	***IMP Year 1, The Pit and the Pendulum:*** Correlation or Cause and Effect?, 222 ***IMP Year 2, The Game of Pig:*** Coincidence or Causation?, 205 ***IMP Year 3, Is There Really a Difference?:*** Late in the Day, 475
Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments	
1. Understand that statistics is a process for making inferences about population parameters based on a random sample from that population.	***IMP Year 1, The Pit and the Pendulum:*** Pendulum Variations, 182 ***IMP Year 3, Is There Really a Difference?*** Two Different Differences, 435
2. Decide if a specified model is consistent with results from a given data-generating process, e.g. using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	***IMP Year 2, The Game of Pig:*** Waiting for a Double, 201 Expecting the Unexpected, 204 ***IMP Year 3, Is There Really a Difference?:*** Loaded or Not?, 442
Make inferences and justify conclusions from sample surveys, experiments and observational studies	
3. Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each.	***IMP Year 3, Is There Really a Difference?:*** Samples and Populations, 429 Who Gets A's and Measles?, 432
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	***IMP Year 3, Is There Really a Difference?:*** Try This Case, 431 Fair Dice, 441
5. Use data from a randomized experiment to compare two treatments; justify significant differences between parameters through the use of simulation models for random assignment.	***IMP Year 3, Is There Really a Difference?:*** Loaded or Not?, 442 The Spoon or the Coin?, 456 Random but Fair, 469

STATISTICS AND PROBABILITY	
Standard	***IMP* Lessons**
6. Evaluate reports based on data.	***IMP Year 3, Is There Really a Difference?:*** Bad Research, 490 On Tour with χ^2, 491
Conditional Probability and the Rules of Probability	
Understand independence and conditional probability and use them to interpret data	
1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	***IMP Year 2, The Game of Pig:*** What Are the Chances?, 206 Rug Games, 210 Portraits of Probabilities, 211
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	***IMP Year 2, The Game of Pig:*** Mystery Rugs, 215 Martian Basketball, 229
3. Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	***IMP Year 2, The Game of Pig:*** The Theory of One-and-One, 225
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your class will favor science given that the student is a boy. Do the same for other subjects and compare the results.*	***IMP Year 3, Is There Really a Difference?:*** Samples and Populations, 429 POW 16: A Difference Investigation, 473

Statistics and Probability	
Standard	***IMP* Lessons**
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of being unemployed if you are female with the chance of being female if you are unemployed.*	***IMP Year 3, Is There Really a Difference?*** Quality of Investigation, 433 Who Gets A's and Measles?, 432
Use the rules of probability to compute probabilities of compound events in a uniform probability model	
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.	***IMP Year 1, The Game of Pig*** Streak-Shooting Shelly, 226 Little Pig Strategies, 233
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	***IMP Year 2, The Game of Pig:*** One-and-One, 223 Streak-Shooting Shelly, 226 A Sixty Percent Solution, 234
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	***IMP Year 2, The Game of Pig:*** Paula's Pizza, 249
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	***IMP Year 3, Pennant Fever:*** Who's on First?, 44 Five for Seven, 46 What's for Dinner?, 49
Using Probability to Make Decisions	
Calculate expected values and use them to solve problems	
1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	***IMP Year 2, The Game of Pig:*** Pointed Rugs, 220 The Theory of One-and-One, 225 The Theory of Two-Dice Sums, 253
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	***IMP Year 2, The Game of Pig:*** Waiting for a Double, 201 Expecting the Unexpected, 204 Rollin', Rollin', Rollin', 217

STATISTICS AND PROBABILITY

Standard	*IMP* Lessons
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*	***IMP Year 2, The Game of Pig:*** The Game of Pig, 196 Little Pig Strategies, 233 Pig Tails Decisions, 278
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	***IMP Year 1: The Pit and the Pendulum:*** Standard Pendulum Data and Decisions, 181 Pendulum Variations, 182 ***IMP Year 3, Is There Really a Difference?:*** Delivering Results, 483 Paper or Plastic?, 485 *Two Different Differences* Revisited, 487
Use probability to evaluate outcomes of decisions	
5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	***IMP Year 2, The Game of Pig:*** Spinner Give and Take, 219
5a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	***IMP Year 2, The Game of Pig:*** Mia's Cards, 221 Aunt Zena at the Fair, 280 The Lottery and Insurance—Why Play?, 228
5b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	***IMP Year 2, The Game of Pig:*** A Fair Rug Game?, 222 Simulating the Carrier, 275 Another Carrier Dilemma, 276
6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	***IMP Year 2, The Game of Pig:*** A Fair Rug Game, 222 Spins and Draws, 227

Statistics and Probability	
Standard	***IMP* Lessons**
7. Analyze decisions and strategies using probability concepts (e.g. product testing, medical testing, pulling a hockey goalie at the end of a game).	***IMP Year 2, The Game of Pig:*** A Fair Deal for the Carrier?, 274